

Possibility of observing Leptonic CP violation with perturbed Democratic mixing patterns

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Abstract

The Daya Bay oscillation has recently reported the precise measurement of $\theta_{13} \simeq 8.8^\circ \pm 0.8^\circ$ or $\theta_{13} \neq 0$ at 5.2σ level. The observed non-zero θ_{13} can be accommodated by some general modifications to the Democratic mixing matrix. Using such matrices we study the possibility of observing non-zero CP violation in the leptonic sector.

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I. INTRODUCTION

The observation of neutrino oscillations has revealed that neutrinos may have non-zero masses and lepton flavors are mixed. Thus, analogous to the quark mixing, the three flavor eigenstates of neutrinos (ν_e, ν_μ, ν_τ) are related to the corresponding mass eigenstates (ν_1, ν_2, ν_3) by the unitary transformation

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \quad (1)$$

where V is the 3×3 unitary matrix known as PMNS matrix [1, 2], which contains three mixing angles and three CP violating phases (one Dirac two Majorana phases). The mixing matrix V can be parameterized in terms of three mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$ and three CP-violating phases δ, ρ, σ [3] as

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} P_\nu, \quad (2)$$

where $c_{ij} \equiv \cos \theta_{ij}$, $s_{ij} \equiv \sin \theta_{ij}$ and $P_\nu \equiv \{e^{i\rho}, e^{i\sigma}, 1\}$ is a diagonal matrix with CP violating Majorana phases ρ and σ . The global analysis of the recent results of various neutrino oscillation experiments [4] suggest the neutrino masses and mixing parameters at 1σ level to be

$$\begin{aligned} \Delta m_{21}^2 &= 7.59_{-0.18}^{+0.20} \times 10^{-5} \text{eV}^2, \\ \Delta m_{31}^2 &= \begin{cases} + (2.50_{-0.16}^{+0.09}) \times 10^{-3} \text{ eV}^2 & \text{for normal hierarchy (NH)} \\ - (2.40_{-0.09}^{+0.08}) \times 10^{-3} \text{ eV}^2 & \text{for inverted hierarchy (IH)} \end{cases} \\ \sin^2 \theta_{12} &= 0.312_{-0.015}^{+0.017}, \quad \sin^2 \theta_{23} = 0.52 \pm 0.06, \\ \sin^2 \theta_{13} &= 0.013_{-0.005}^{+0.007} \quad (\text{NH}) \quad \sin^2 \theta_{13} = 0.016_{-0.006}^{+0.008} \quad (\text{IH}). \end{aligned} \quad (3)$$

Furthermore, evidence for $\theta_{13} \neq 0^\circ$ at about 3σ level has been obtained in a global analysis [5]. In 2011, data from T2K [6], MINOS [7] and Double Chooz experiment [8] ruled out, for the first time, $\theta_{13} = 0$ at 3σ level.

The Daya Bay Collaboration [9] has recently reported the first precise measurement of θ_{13} from the reactor $\bar{\nu}_e \rightarrow \bar{\nu}_e$ oscillations. The best fit (1σ) result is

$$\sin^2 2\theta_{13} = 0.092 \pm 0.016(\text{stat}) \pm 0.005(\text{syst}), \quad (4)$$

which is equivalent to $\theta_{13} \simeq 8.8^\circ \pm 0.8^\circ$ or $\theta_{13} \neq 0$ at 5.2σ level. This is followed by the results from RENO collaboration [10]

$$\sin^2 2\theta_{13} = 0.113 \pm 0.013(\text{stat}) \pm 0.019(\text{syst}). \quad (5)$$

These exciting observations imply that the smallest neutrino mixing angle is not really small and the PMNS mixing matrix V is not strongly hierarchical. Evidence of non-zero reactor angle θ_{13} yields a potentially measurable CP phase δ in future neutrino oscillation experiments. Thus, one of the important implications of such observation is that leptonic CP violation could be observable, analogous to the observed CP violation in the quark sector.

The purpose of the present paper is to look for the possible existence of CP violation in the lepton sector. The strength of CP violation in neutrino oscillations is described by the Jarlskog rephasing invariant [11]

$$J = \text{Im}(V_{e1}V_{\mu 2}V_{e2}^*V_{\mu 1}^*) = \text{Im}(V_{e2}V_{\mu 3}V_{e3}^*V_{\mu 2}^*) = \cdots = c_{12}s_{12}c_{13}^2s_{13}c_{23}s_{23}\sin\delta, \quad (6)$$

which is proportional to the sine of the smallest mixing angle θ_{13} . In the quark sector the corresponding Jarlskog invariant is found to be $J_q \sim \mathcal{O}(10^{-5})$ which is attributed to the strongly suppressed values of the quark flavor mixing angles. In the lepton sector, since there are two large mixing angles, it could be possible to achieve a relatively large J , if the CP violating phase is not vanishingly small.

II. METHODOLOGY

It has been shown in Ref. [12] that perturbations to various well-known mixing patterns, i.e., bimaximal (BM) [13–18], tri-bimaximal (TB) [19–28] and democratic mixing pattern (DC) [29, 30], would lead to neutrino mixing angles as given by current neutrino experiments. However, in Ref. [31] it is very elaborately discussed that out of the five typical mixing patterns, i.e., the democratic, bimaximal, tri-bimaximal, golden ratio and hexagonal forms, the democratic mixing pattern provides a more natural perturbation matrix, which can be obtained easily from either the flavour symmetry breaking or quantum corrections. For a more clear illustration let us consider the mixing matrix to have the form

$$V = (V_0 + \Delta V)P_\nu \quad (7)$$

in which the leading term V_0 is a constant matrix responsible for two larger mixing angles θ_{12} and θ_{23} and the correction term ΔV is a perturbation matrix responsible for both the smallest mixing angle θ_{13} and the Dirac CP violating phase δ . Considering V_0 to be the Democratic mixing matrix V_{DC} :

$$V_{\text{DC}} = \begin{pmatrix} \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & 0 \\ \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{6}} & -\sqrt{\frac{2}{3}} \\ -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{3}} \end{pmatrix}. \quad (8)$$

the three mixing angles are found to be $\theta_{12}^{(0)} = 45^\circ$, $\theta_{13}^{(0)} = 0^\circ$ and $\theta_{23}^{(0)} = \arctan(\sqrt{2}) \simeq 54.7^\circ$. Therefore, the corrections to all the large mixing angles θ_{12} and θ_{23} are found to be

$$\theta^* \equiv \theta_{12}^{(0)} - \theta_{12} = \theta_{23}^{(0)} - \theta_{23} \approx 10^\circ. \quad (9)$$

This value of θ^* is quite interesting as it is very close the observed non-zero θ_{13} .

Motivated by the success of the DC mixing matrix in explaining the nonzero θ_{13} , we would like to scrutinize further the implication of this mixing pattern. It is well known that the Democratic mass matrix is one of the most interesting candidate for the texture of quark and charged-lepton mass matrices, since it naturally explains why the third generation particles are much heavier than the first two generations. The democratic mixing matrix V_{DC} was originally obtained, as the leading term of the lepton-flavor mixing matrix V_{PMNS} , from the breaking of $S(3)_L \times S(3)_R$ flavor symmetry of the charged lepton mass matrix in the basis where the neutrino mass matrix is diagonal [29, 30]. First we will briefly review the modified DC mixing pattern and the resulting consequences. The general modification of the mixing matrix [12], which will give nonzero θ_{13} could be one of the following forms

$$\begin{aligned} 1. & \quad V_{\text{PMNS}} = V_{\text{DC}} \cdot V_{ij} , \\ 2. & \quad V_{\text{PMNS}} = V_{ij} \cdot V_{\text{DC}} , \\ 3. & \quad V_{\text{PMNS}} = V_{\text{DC}} \cdot V_{ij} \cdot V_{kl} , \\ 4. & \quad V_{\text{PMNS}} = V_{ij} \cdot V_{kl} \cdot V_{\text{DC}} , \end{aligned} \quad (10)$$

where $(ij), (kl) = (12), (13), (23)$ respectively. The perturbation mixing matrices V_{ij} are

given by

$$\begin{aligned}
V_{12} &= \begin{pmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{pmatrix}, & V_{23} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos y & \sin y e^{i\delta'} \\ 0 & -\sin y e^{-i\delta'} & \cos y \end{pmatrix} \\
V_{13} &= \begin{pmatrix} \cos z & 0 & \sin z e^{i\delta'} \\ 0 & 1 & 0 \\ -\sin z e^{-i\delta'} & 0 & \cos z \end{pmatrix}. & & (11)
\end{aligned}$$

In Ref. [12], it has been shown that out of the eighteen possible forms only five will accommodate the observed neutrino oscillation data. These five forms are listed below:

- i.* $V_{\text{DC}}V_{13}V_{12}$
 - ii.* $V_{\text{DC}}V_{23}V_{13}$
 - iii.* $V_{23}V_{13}V_{\text{DC}}$
 - iv.* $V_{23}V_{12}V_{\text{DC}}$
 - v.* $V_{13}V_{12}V_{\text{DC}}$
- (12)

The implications of these five forms are extensively studied in Ref. [32], if one considers texture one-zero mass matrices with vanishing CP violating phases. In this paper we would like to investigate in detail these mixing patterns and their implications towards the observation of CP violation in the neutrino sector without assuming the Dirac type CP violating phase to be zero.

A. Case 1: $V = V_{\text{DC}}V_{13}V_{12}$

Now we will consider the case where the PMNS matrix takes the form $V = V_{\text{DC}}V_{13}V_{12}$, which yields

$$V = \begin{pmatrix} \frac{c_x c_z - s_x}{\sqrt{2}} & \frac{c_x + c_z s_x}{\sqrt{2}} & \frac{s_z}{\sqrt{2}} e^{i\delta'} \\ \frac{s_x}{\sqrt{6}} + c_x \left(\frac{c_z}{\sqrt{6}} + \sqrt{\frac{2}{3}} s_z e^{i\delta'} \right) & \frac{-c_x}{\sqrt{6}} + s_x \left(\frac{c_z}{\sqrt{6}} + \sqrt{\frac{2}{3}} s_z e^{i\delta'} \right) & -\sqrt{\frac{2}{3}} c_z + \frac{s_z}{\sqrt{6}} e^{i\delta'} \\ -\frac{s_x}{\sqrt{3}} + c_x \left(-\frac{c_z}{\sqrt{3}} + \frac{s_z}{\sqrt{3}} e^{i\delta'} \right) & \frac{c_x}{\sqrt{3}} + s_x \left(-\frac{c_z}{\sqrt{3}} + \frac{s_z}{\sqrt{3}} e^{i\delta'} \right) & -\frac{c_z}{\sqrt{3}} - \frac{s_z}{\sqrt{3}} e^{i\delta'} \end{pmatrix}, \quad (13)$$

By comparing the above matrix with the standard PMNS matrix (1) one can obtain the values of the perturbation parameters x and z . For illustration let us compare the (1,3)

element of both the matrices which gives us

$$s_{13}e^{-i\delta} = \frac{s_z}{\sqrt{2}}e^{i\delta'} . \quad (14)$$

Taking the modulus on both sides one can obtain the value of z . For numerical analysis we will use the the values of mixing angles from [4, 9] as

$$\sin^2 \theta_{12} = 0.312_{-0.015}^{+0.017}, \quad \sin^2 \theta_{23} = 0.52 \pm 0.06, \quad \sin^2 \theta_{13} = 0.023 \pm 0.004. \quad (15)$$

Using these input parameters, one can obtain obtain $z = (12.5 \pm 1.1)^\circ$. Similarly one can obtain the value of x from the ratio of the elements $V(1, 2)$ and $V(1, 1)$ as

$$\tan x = \frac{c_z \tan \theta_{12} - 1}{c_z + \tan \theta_{12}} , \quad (16)$$

which gives us $x = (-11.71 \pm 0.12)^\circ$. To constrain the Dirac-type CP violating phase, we take the ratio of $V(2, 3)$ and $V(3, 3)$ elements, which gives

$$\cos \delta = \frac{c_z(\sqrt{2} - \tan \theta_{23})}{\sin \theta_{13}(1 + \sqrt{2} \tan \theta_{23})} \quad (17)$$

Now varying the value of the mixing angle θ_{23} within 1σ range, we show in Figure-1 the correlation plot between the Dirac type CP violating phase δ and θ_{13} . From the figure it can be seen that the allowed range of δ will be between $(0 - 60)^\circ$.

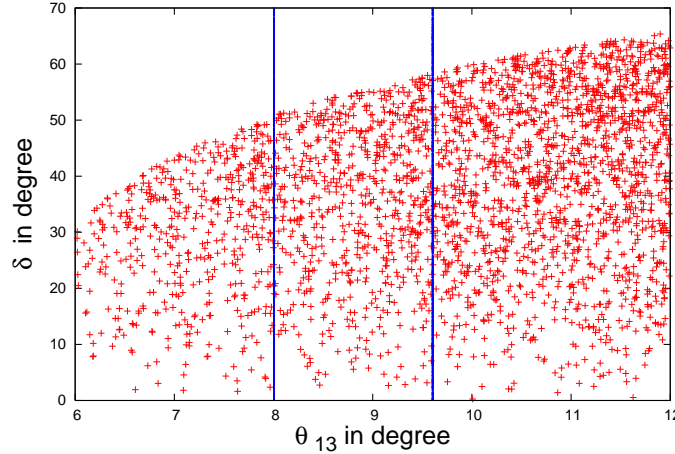


FIG. 1: The correlation plot between the mixing angle θ_{13} and Dirac CP violating phase δ . The vertical lines represent the $1 - \sigma$ allowed range of θ_{13} .

The Jarlskog invariant is found to be

$$J = \frac{1}{3\sqrt{2}}s_z(c_{2x}c_z - s_{2x}\frac{s_z^2}{2})\sin \delta' = \frac{1}{3}s_{13}(c_{2x}c_z - s_{2x}\frac{s_z^2}{2})\sin \delta = (0.046 \pm 0.004)\sin \delta . \quad (18)$$

Thus, substituting the value of δ as obtained from the correlation plot, we expect the CP violation parameter J could be $J \leq 0.04$.

Now to evaluate the effective electron neutrino mass m_{ee} that appears in neutrino less double beta decay, we work in the basis where the charged leptons are diagonal and extract the (1,1) matrix element of the rotated neutrino mass matrix given as [32]

$$m_{ee} = |m_1 V_{11}^2 + m_2 V_{12}^2 + m_3 V_{13}^2| . \quad (19)$$

Assuming normal hierarchy structure of neutrino masses, we can eliminate the heavier neutrino masses m_2 and m_3 in terms of the lightest neutrino mass m_1 and the observed mass square differences as

$$\begin{aligned} m_2 &= \sqrt{m_1^2 + \Delta m_{21}^2} \\ m_3 &= \sqrt{m_1^2 + \Delta m_{31}^2} , \end{aligned} \quad (20)$$

we show in Figure-2, the variation of m_{ee} with m_1 , where the other parameters are allowed to vary within their $1 - \sigma$ range. Thus, for m_1 below $\mathcal{O}(10^{-2})$ eV, we get $m_{ee} \leq 3.4 \times 10^{-2}$ eV.

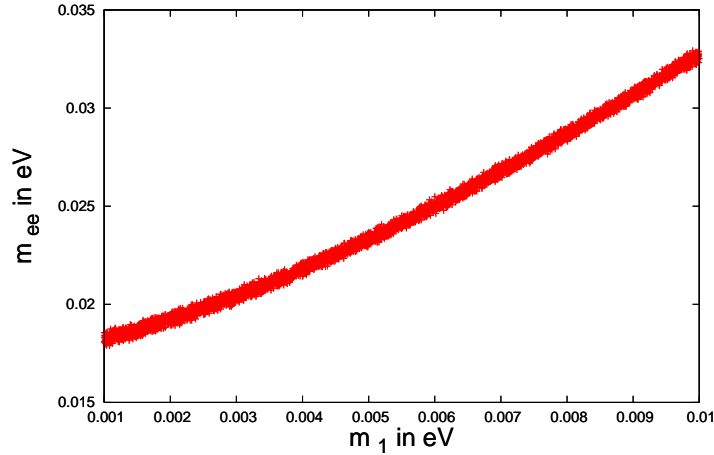


FIG. 2: Variation of m_{ee} with the lightest neutrino mass m_1 for the NH scenario.

B. Case 2: $V = V_{\text{DC}}V_{23}V_{13}$

Here we will consider the mixing matrix of the form $V = V_{\text{DC}}V_{23}V_{13}$, which gives

$$V = \begin{pmatrix} \frac{1}{\sqrt{2}}(c_z - s_y s_z e^{2i\delta'}) & \frac{c_y}{\sqrt{2}} & \left(\frac{c_z s_y + s_z}{\sqrt{2}}\right) e^{i\delta'} \\ \frac{c_z}{\sqrt{6}} + s_z e^{i\delta'} \left(\sqrt{\frac{2}{3}}c_y + \frac{s_y}{\sqrt{6}}e^{i\delta'}\right) & -\frac{c_y}{\sqrt{6}} + \sqrt{\frac{2}{3}}s_y e^{i\delta'} & -c_z \left(\sqrt{\frac{2}{3}}c_y + \frac{s_y}{\sqrt{6}}e^{i\delta'}\right) + \frac{s_z}{\sqrt{6}}e^{i\delta'} \\ -\frac{c_z}{\sqrt{3}} - s_z \left(-\frac{c_y}{\sqrt{3}} + \frac{s_y}{\sqrt{3}}e^{i\delta'}\right) e^{i\delta'} & \frac{1}{\sqrt{3}}(c_y + s_y e^{i\delta'}) & c_z \left(-\frac{c_y}{\sqrt{3}} + \frac{s_y}{\sqrt{3}}e^{i\delta'}\right) - \frac{s_z}{\sqrt{3}}e^{i\delta'} \end{pmatrix} \quad (21)$$

Comparing (1,2) element of above matrix with that of (2) we obtain

$$\cos y = \sqrt{2} \sin \theta_{12} \cos \theta_{13} . \quad (22)$$

Substituting the values for θ_{12} and θ_{13} in (22) we obtain $y = (38.66 \pm 0.15)^\circ$. Similarly comparing the (1,3) element of (2) and (21), and doing some algebraic manipulation we obtain $z = -(21.5 \pm 0.8)^\circ$.

Proceeding in the same manner as in the previous case we obtain the CP violating phase δ and s_{13} as

$$\cos \delta = \left(\frac{c_z s_y + s_z}{c_z s_y - s_z} \right) \left(\frac{\tan(\theta_{23}) - \sqrt{2}}{\sqrt{2} \tan(\theta_{23}) + 1} \right) \frac{c_y c_z}{s_{13}} . \quad (23)$$

Varying the parameters within their $1 - \sigma$ range, the correlation plot between δ and s_{13} is shown in Figure-3, which shows that the allowed range of δ in this case is between $(96 - 105)^\circ$. The expression for CP-violation parameter J is found to be

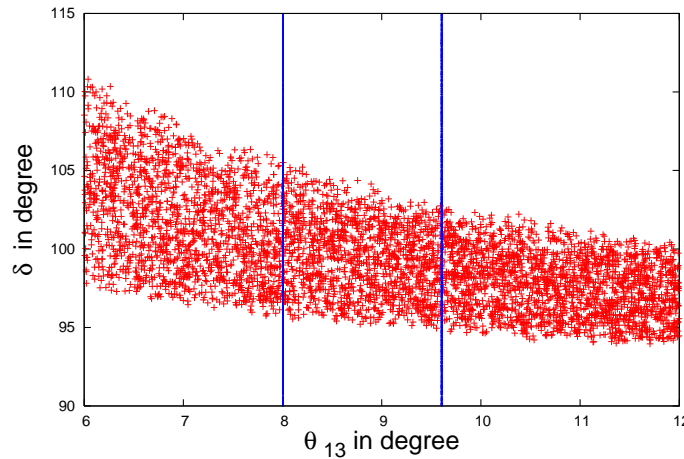


FIG. 3: The correlation plot between the mixing angle θ_{13} and Dirac CP violating phase δ . The vertical lines represent the $1 - \sigma$ allowed range of θ_{13} .

$$J = \sin \delta' \left(\frac{-s_y^2 c_y s_z c_z}{6} + \frac{c_y^2 s_y s_z^2}{6} - \frac{s_y^3 c_y s_z^2}{6} \right) - \sin 2\delta' \left(\frac{s_y^2 c_y^2 s_z^2}{3} \right), \quad (24)$$

where δ' is related to δ through

$$\cos \delta' = \left(\frac{\sqrt{2}s_{13}}{c_z s_y + s_z} \right) \cos \delta. \quad (25)$$

Now substituting y and z values in (24) we obtain $J \approx (0.024 - 0.027)$.

C. Case 3: $V = V_{23}V_{13}V_{DC}$

In this case the mixing matrix V is found to be

$$V = \begin{pmatrix} \frac{c_z}{\sqrt{2}} - \frac{s_z}{\sqrt{3}}e^{i\delta'} & \frac{c_z}{\sqrt{2}} + \frac{s_z}{\sqrt{3}}e^{i\delta'} & -\frac{s_z}{\sqrt{3}}e^{i\delta'} \\ \frac{c_y - \sqrt{2}c_z s_y e^{i\delta'} - \sqrt{3}s_y s_z e^{2i\delta'}}{\sqrt{6}} & \frac{-c_y + \sqrt{2}c_z s_y e^{i\delta'} - \sqrt{3}s_y s_z e^{2i\delta'}}{\sqrt{6}} & \frac{-\sqrt{2}c_y - c_z s_y e^{i\delta'}}{\sqrt{3}} \\ -\frac{\sqrt{2}c_y c_z + s_y e^{i\delta'} + \sqrt{3}c_y s_z e^{i\delta'}}{\sqrt{6}} & \frac{\sqrt{2}c_y c_z + s_y e^{i\delta'} - \sqrt{3}c_y s_z e^{i\delta'}}{\sqrt{6}} & \frac{-c_y c_z + \sqrt{2}e^{i\delta'} s_y}{\sqrt{3}} \end{pmatrix}. \quad (26)$$

As done in the previous cases from the confrontation of the elements of (2) with (26) we obtain the perturbation angles from

$$s_{13}e^{-i\delta} = \frac{s_z}{\sqrt{3}}e^{i\delta'} \quad (27)$$

$$\tan y = \frac{(c_z \tan \theta_{23} - \sqrt{2})}{(c_z - \sqrt{2} \tan \theta_{23})} \quad (28)$$

$z = (15.22_{-1.14}^{+1.32})^\circ$ and $y = (38.98 \pm 0.10)^\circ$.

Expression for CP-violation phase is obtained by comparing the ratio of (2,3) and (3,3) elements of (2) and (26) and replacing $\cos \delta'$ by

$$\cos \delta' = \frac{\sqrt{3}s_{13} \cos \delta}{s_z}, \quad (29)$$

as

$$\cos \delta = \frac{s_z c_y \tan \theta_{23} (c_z - \sqrt{2})}{\sqrt{3}s_y (\sqrt{2} \tan \theta_{23} + c_z)} \frac{1}{s_{13}}. \quad (30)$$

In this case the allowed range of δ is found to be $(100 - 110)^\circ$.

The Jarlskog invariant parameter is found to be

$$J = \sin \delta' \left(\frac{c_y^2 s_z c_z}{3\sqrt{6}} - \frac{\sqrt{2}}{3} c_z s_z^2 c_y s_y - \frac{s_z^2 c_y s_y c_z}{9\sqrt{2}} \right) - \sin 2\delta' \left(\frac{c_z^2 s_z c_y s_y}{3\sqrt{3}} - \frac{s_y^2 s_z^2 c_z^2}{6} \right) + \sin 3\delta' \left(\frac{s_z^2 c_z s_y c_y}{3\sqrt{2}} \right), \quad (31)$$

which yields $J = \mathcal{O}(10^{-3})$.

D. Case 4: $V = V_{23}V_{12}V_{DC}$

For $V = V_{23}V_{12}V_{DC}$ form, we get

$$V = \begin{pmatrix} \frac{c_x}{\sqrt{2}} + \frac{s_x}{\sqrt{6}} & \frac{c_x}{\sqrt{2}} - \frac{s_x}{\sqrt{6}} & -\sqrt{\frac{2}{3}}s_x \\ \frac{c_x c_y - \sqrt{3}c_y s_x - \sqrt{2}s_y e^{i\delta'}}{\sqrt{6}} & \frac{-c_x c_y - \sqrt{3}c_y s_x + \sqrt{2}s_y e^{i\delta'}}{\sqrt{6}} & -\frac{\sqrt{2}c_x c_y + s_y e^{i\delta'}}{\sqrt{3}} \\ \frac{-\sqrt{2}c_y - c_x s_y e^{i\delta'} + \sqrt{3}s_x s_y e^{i\delta'}}{\sqrt{6}} & \frac{\sqrt{2}c_y + c_x s_y e^{i\delta'} + \sqrt{3}s_x s_y e^{i\delta'}}{\sqrt{6}} & \frac{-c_y + \sqrt{2}c_x s_y e^{i\delta'}}{\sqrt{3}} \end{pmatrix}. \quad (32)$$

In this case the perturbation angles are found to be $x = (10.7 \pm 1.0)^\circ$ and $y = -(7.85_{-0.13}^{+0.08})^\circ$ and one can obtain a correlation between θ_{13} and θ_{12} as

$$\sin \theta_{13} = \frac{1 - \tan \theta_{12}}{\sqrt{2(1 - \tan \theta_{12} + \tan^2 \theta_{12})}}. \quad (33)$$

In this scenario, if we allow θ_{12} to vary in its 3σ range a very narrow parameter space for θ_{13} can be found in the $\theta_{12} - \theta_{13}$ plane. The Jarlskog Invariant is found to be

$$J = -\sqrt{\frac{2}{3}}s_y c_y s_x \left(\frac{c_x^2}{2} - \frac{s_x^2}{6} \right) \sin \delta' = ((9.8 \pm 0.7) \times 10^{-3}) \sin \delta'. \quad (34)$$

However, it is not possible to constrain the CP violating phase δ in this case.

E. Case 5 : $V = V_{13}V_{12}V_{DC}$

Here the mixing matrix is given as

$$V = \begin{pmatrix} \frac{\sqrt{3}c_x c_z + c_z s_x - \sqrt{2}s_z e^{i\delta'}}{\sqrt{6}} & \frac{\sqrt{3}c_x c_z - c_z s_x + \sqrt{2}s_z e^{i\delta'}}{\sqrt{6}} & -\frac{\sqrt{2}c_z s_x + s_z e^{i\delta'}}{\sqrt{3}} \\ \frac{c_x}{\sqrt{6}} - \frac{s_x}{\sqrt{2}} & -\frac{c_x}{\sqrt{6}} - \frac{s_x}{\sqrt{2}} & -\sqrt{\frac{2}{3}}c_x \\ \frac{-\sqrt{2}c_z + \sqrt{3}c_x s_z e^{i\delta'} + s_x s_z e^{i\delta'}}{\sqrt{6}} & \frac{\sqrt{2}c_z - \sqrt{3}c_x s_z e^{i\delta'} + s_x s_z e^{i\delta'}}{\sqrt{6}} & \frac{-c_z + \sqrt{2}s_x s_z e^{i\delta'}}{\sqrt{3}} \end{pmatrix}. \quad (35)$$

The perturbation angles are $x = (151.3 \pm 0.2)^\circ$ and $z = (28.8 \pm 0.1)^\circ$ obtained from

$$-\sqrt{\frac{2}{3}}c_x = c_{13}s_{23}, \quad (36)$$

and

$$\tan z = \frac{\sqrt{3}c_x \tan \theta_{12} + s_x \tan \theta_{12} - \sqrt{3}c_x + s_x}{\sqrt{2}(1 + \tan \theta_{12})}. \quad (37)$$

As discussed in Ref. [12], this scenario also does not work for the best fit values and is valid only for a narrow region in the $\theta_{12} - \theta_{23}$ plane. The Jarlskog invariant is found to be

$$J = \left(\frac{c_x c_z s_z}{\sqrt{6}} - \frac{s_z c_z s_x}{\sqrt{2}} \right) \left(\frac{c_x^2}{3} - \frac{c_x s_x}{\sqrt{3}} \right) \sin \delta' = -(0.147 \pm 0.0007) \sin \delta'. \quad (38)$$

Here also it is not possible to obtain the bounds on the CP violating phase δ .

III. SUMMARY AND CONCLUSION

Among the three mixing angles of the neutrino mixing matrix, the smallest reactor angle θ_{13} is the most important one to understand the lepton mixing pattern completely. One of the main objective of the currently running and upcoming neutrino experiments is to measure it very precisely. The recent results from T2K, MINOS, Double Chooz, Daya Bay and RENO experiments indicate non-zero and relatively largish θ_{13} . The fact that θ_{13} is not strongly suppressed is certainly a good news to the experimental attempts towards measurement of CP violation in the lepton sector. Motivated by this relatively large value of the reactor mixing angle θ_{13} , we have studied the possibility of observing leptonic CP violation for a class of modified democratic mixing patterns in a systematic way. It has been shown that a largish θ_{13} can be accommodated by a general modification of the democratic mixing matrix. The perturbations are of the form of Euler rotation angles and the perturbation parameters can be determined by using the known values of the neutrino mixing angles. If the neutrino mass matrix satisfies texture one-zero pattern, then out of the eighteen such possible modifications only five are physically allowed. We consider those five modified DC mixing matrices and study their implications on possible observation of CP violation in the neutrino sector. We also obtain some non-trivial correlation between θ_{13} and the Dirac CP violating phase δ . We have also estimated the leptonic CP violation as well as the effective electron neutrino mass m_{ee} which involves in neutrino less double beta decay process. Our result indicates that it could be possible to observe such CP violation effect in the upcoming long baseline experiments.

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